## CS565: Intelligent Systems and Interfaces



Lecture: Language Modeling
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Ashish Anand
IIT Guwahati

## Recap and Moving Forward

- Until Now
- Sentence segmentation, Tokenization
- Collocation
- Next
- Language Modeling: Generative model of language


## Objective

- Understanding Language Model
- N-Gram Language Model
- Evaluating Language Model


## Lets look at some examples

- Predicting next word
- I am planning
- Speech Recognition
- I saw a van vs I saw a 7


## Example continued

- Spelling correction
- Study was conducted by students vs study was conducted $\underline{b e}$ students
- Their are two exams for this course vs There are two exams for this course
- Machine Translation
- I have asked him to do homework
- मैंने उससे पूछा कि होमवर्क करने के लिए
- मैंने उसे होमवर्क करने के लिए कहा


## My objective was

- To find next probable word
- To find which sentence is more likely to be true

But it must be recognized that the notion "probability of a sentence" is an entirely useless one, under any known interpretation of this term.

Noam Chomsky

Anytime a linguist leaves the group the recognition rate goes up.
Fred Jelinek (then of the IBM speech group)

## Language Models (LM)

- Models assigning probabilities to a sequence of words
- $P($ I saw a van $)>P($ l saw a 7$)$
-P(मैंने उससे पूछा कि होमवर्क करने के लिए) < P(मैंने उसे होमवर्क करने के लिए कहा)


## Defining LM Formally

- We consider a vocabulary, a finite set denoted as $\mathcal{V}$, and a function $P\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)$, such that
- For any $\left\langle w_{1}, w_{2}, w_{3}, \ldots ., w_{n}\right\rangle \in \mathcal{V}^{\dagger}, p\left(w_{1}, w_{2}, w_{3}, \ldots ., w_{n}\right) \geq$ 0
- $\Sigma p\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)=1$,
where $\mathcal{V}^{\dagger}:\left\{S=w_{1} w_{2} w_{3} \ldots w_{n} \mid w_{i} \in \mathcal{V}\right\}$.


## Estimating $P\left(w_{1}, w_{2}, . ., w_{n}\right)$

- Our task is to compute

P(I, am, fascinated, with, recent, advances, in, AI)

- Chain Rule


## Estimating $\mathrm{P}\left(\mathrm{w}_{1}, \mathrm{w}_{2}, . ., \mathrm{w}_{\mathrm{n}}\right)$

- Chain Rule
- $P\left(w_{1}, w_{2}, w_{3}, \ldots ., w_{n}\right)=P\left(w_{1}\right) P\left(w_{2} \mid w_{1}\right) P\left(w_{3} \mid w_{1}, w_{2}\right) \ldots$. $\mathrm{P}\left(\mathrm{w}_{\mathrm{n}} \mid \mathrm{w}_{1}, . ., \mathrm{w}_{\mathrm{n}-1}\right)$


## Estimating $\mathrm{P}\left(\mathrm{w}_{\mathrm{n}} \mid \mathrm{w}_{1}, . ., \mathrm{w}_{\mathrm{n}-1}\right)$

- Could we just count and divide?

$$
P(e a t \mid I \text { want to })=\frac{\operatorname{count}(I \text { want to eat })}{\operatorname{count}(\text { I want to })}
$$

- What is the problem here?


## Estimating $\mathrm{P}\left(\mathrm{w}_{\mathrm{n}} \mid \mathrm{w}_{1}, . ., \mathrm{w}_{\mathrm{n}-1}\right)$

- Too many possible sentences
- Data sparseness
- Poor generalizability


## Markov Assumption

- Simplifying assumption:

$$
P(\text { eat } \mid I \text { want to }) \sim P(\text { eat } \mid \text { to })
$$

or

$$
P(\text { eat } \mid I \text { want to }) \sim P(\text { eat } \mid \text { want to })
$$

## Markov Assumption

$$
P\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right) \sim \prod_{i} P\left(w_{i} \mid w_{i-k}, \ldots ., w_{i-1}\right)
$$

i.e., Each component in the product is getting approximated by Markov assumption

$$
P\left(w_{i} \mid w_{1}, w_{2}, w_{3}, \ldots ., w_{i-1}\right) \sim P\left(w_{i} \mid w_{i-k}, \ldots ., w_{i-1}\right)
$$

## N-gram Models

- Unigram: Simplest Model (does not depend on anything)

$$
P\left(w_{1}, w_{2}, w_{3}, \ldots ., w_{n}\right) \sim \prod_{i} P(w i)
$$

- Bigram Model ( $1^{\text {st }}$ Order Markov model)

$$
P\left(w_{1}, w_{2}, w_{3}, \ldots ., w_{n}\right) \sim \prod_{i} P\left(w_{i} \mid w_{i-1}\right)
$$

- Trigram Model (2 ${ }^{\text {nd }}$ order Markov model)

$$
P\left(w_{1}, w_{2}, w_{3}, \ldots ., w_{n}\right) \sim \prod_{i} P\left(w_{i} \mid w_{i-2}, w_{i-1}\right)
$$

## N-gram Model: Issue

- Long-distance dependencies
"The computer which I had just put into the lab on the fifth floor crashed"


## Estimating the Probabilities

## Data

- Training
- Development
- Test


## Maximum Likelihood Estimate

- Unigram

$$
P\left(w_{i}\right)=\frac{c\left(w_{i}\right)}{K} ;
$$

K: Total number of tokens in training set

- Bigram

$$
P\left(w_{i} \mid w_{i-1}\right)=\frac{c\left(w_{i-1}, w_{i}\right)}{c\left(w_{i-1}\right)}
$$

- N-Gram

$$
P\left(w_{n} \mid w_{n-N+1}^{n-1}\right)=\frac{c\left(w_{n-N+1}^{n-1} w_{n}\right)}{c\left(w_{n-N+1}^{n-1}\right)}
$$

## Bigram Probabilities

| eat on | .16 | eat Thai | .03 |
| :--- | :--- | :--- | :--- |
| eat some | .06 | eat breakfast | .03 |
| eat lunch | .06 | eat in | .02 |
| eat dinner | .05 | eat Chinese | .02 |
| eat at | .04 | eat Mexican | .02 |
| eat a | .04 | eat tomorrow | .01 |
| eat Indian | .04 | eat dessert | .007 |
| eat today | .03 | eat British | .001 |

A fragment of bigram probabilities from the Berkeley Restaurant Project showing most likely word to follow eat

## Computing probability of a sentence

| <S> I . 25 | I want . 32 | to . 65 | to eat . 26 | British food . 60 |
| :---: | :---: | :---: | :---: | :---: |
| <s> I'd . 06 | I would . 29 | want a . 05 | to have .14 | British restaurant . 15 |
| <s> Tell . 04 | I don't . 08 | want some . 04 | to spend .09 | British cuisine 01 |
| <s> I'm . 02 | I have . 04 | want thai .01 | to be . 02 | British lunch |

Figure 6.3 More fragments from the bigram grammar from the Berkeley Restaurant Project.
$P(<s>\mid$ want to eat British food $</ s>)=P(I \mid<s>) P($ want |I) $P($ to/want $) P$ (eat/to) P(British/eat) P(food|British) P(</s>|food)

## Language Model Evaluation

Two paradigms

- Intrinsic evaluation
- Extrinsic evaluation


## Intrinsic Evaluation: Perplexity

- Given a test data of $m$ sentences: $s_{1}, s_{2}, \ldots . . ., s_{m}$
- Probability of a sentence under this model $p\left(s_{i}\right)$
- Log-Probability of all sentences: $\log \Pi p\left(s_{i}\right)=\sum \log p\left(s_{i}\right)$


## Perplexity: Alternate definitions

- Perplexity $=2^{-1}$, where $\mathrm{I}=1 / \mathrm{M}\left(\sum \log p\left(s_{i}\right)\right)$
- Perplexity $=P\left(s_{1} s_{2} \ldots \ldots . . s_{n}\right)^{-(1 / M)}$
- Smaller the value of perplexity, better the language model is.


## Interpreting Perplexity

- Weighted average branching factor
- Branching factor: number of possible next words that can follow any word.


## One specific example

- Training: 38 million words from Wall Street Journals [vocab size: 19,979]
- Test: 1.5 million words

|  | Unigram | Bigram | Trigram |
| :--- | :--- | :--- | :--- |
| Perplexity | 962 | 170 | 109 |

## Generalization

- 1 gram: Hill he late speaks; or! a more to leg less first you enter
- 2 gram: What means, sir. I confess she? then all sorts, he is trim, captain
- 3 gram: This shall forbid it should be branded, if renown made it empty
- 4 gram: It cannot be but so.


## Generalization

- 1 gram: Months the my and issue of year foreign .....
- 2 gram: Last December through the way to preserve the Hudson ....
- 3 gram: They also point to ninety nine point six billion dollars from two ......

Source: SLP (3 ${ }^{\text {rd }}$ ed.), Figure 4.4. Training data on 40 million words of Wall Street Journal

## Unknown words

- Fix vocabulary and words within training data not appearing in vocabulary are mapped to <UNK>
- Less frequent words mapped to <UNK>


## Sparsity

- Works well if test corpus is very similar to training, which is not generally the case.
- Training Set
...... denied the allegations denied the reports
...... denied the claims
...... denied the request
- Test Set
.... denied the offer
.... denied the loan
P("offer" | denied the) $=0$

Smoothing

- Next Lecture


## Reference

- SLP (3 $3^{\text {rd }}$ Ed.) , Chapter 4
- Collin's lecture-notes on Language Modeling

