Assignment 2

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CS202: Discrete Mathematics
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September 4, 2017

Question 1. [2 points] Prove the following statement by proving its contrapositive: if r is irrational, then $r^{\frac{1}{5}}$ is irrational.

Question 2. [5 points] Prove that between every two rational numbers, there is an irrational number.

Question 3. [3 points] Show that the argument form with premises $(p \wedge t) \rightarrow (r \vee s)$, $q \rightarrow (u \wedge t)$, $u \rightarrow p$, and $\neg s$, and conclusion $q \rightarrow r$ is valid.

Question 4. [5 points] Show that the union of a countable number of countable sets is countable.

Question 5. [5 points] Show that the set $\mathbb{Z}^+ \times \mathbb{Z}^+$ is countable.

Question 6. [5 points] Show that the set of functions from the positive integers to the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is uncountable.

Question 7. [2+3 points]

I. The square of any integer is either of the form $3k$ or $3k + 1$.

II. Prove that no integer in the following sequence is a perfect square:

$$11,\ 111,\ 1111,\ 11111,$$

Question 8. [5 points] For $n \geq 1$, and $a, b \in \mathbb{Z}^+$, if $a^n | b^n$, then $a | b$.

Question 9. [5points] Find the remainder when the following sum is divided by 4

$$1^5 + 2^5 + 3^5 + \ldots + 99^5 + 100^5$$