Model Sol.[1]

Quiz 3.

(1.)

$$P = \begin{array}{c|ccc|cc} & 2 & 3 & 4 & 1 & 5 \\ \hline 2 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 3 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 4 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \hline 1 & 0 & 0 & 0 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 \end{array}$$

$$Q = \begin{array}{c} 2 \\ 3 \\ 4 \end{array}\begin{bmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{bmatrix} \qquad R = \begin{array}{c|cc} & 1 & 5 \\ \hline 2 & \frac{1}{2} & 0 \\ 3 & 0 & 0 \\ 4 & 0 & \frac{1}{2} \end{array}$$

$$N = (I-Q)^{-1} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}^{-1}$$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & 1 & 0 & 0 \\ -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \to R_2 - (-\frac{1}{2}R_1)} \left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{4} & -\frac{1}{2} & \frac{1}{2} & 1 & 0 \\ 0 & -\frac{1}{2} & 1 & 0 & 0 & 1 \end{array}\right]$$

Subsequent Elimentary op.

$R_3 \to \frac{2}{3}R_2$; $R_3 \to \frac{3}{2}R_3$; $R_2 \to R_2 + \frac{1}{2}R_3$

$R_2 \to \frac{4}{3}R_2$; $R_1 \to R_1 + \frac{1}{2}R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{2} & 1 & \frac{1}{2} \\ 0 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & \frac{1}{2} & 1 & \frac{3}{2} \end{array}\right]$$

ie $N = \begin{bmatrix} \frac{3}{2} & 1 & \frac{1}{2} \\ 1 & 2 & 1 \\ \frac{1}{2} & 1 & \frac{3}{2} \end{bmatrix}$

(a.)

N_{ij} : Expected # of times the chain is in state s_j given that initial state was s_i

Nc, where $c = (1,1,1)^T$

gives expected # of times the chain is in transient states before getting absorbed.

$$\begin{bmatrix} \frac{3}{2} & 1 & \frac{1}{2} \\ 1 & 2 & 1 \\ \frac{1}{2} & 1 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ \boxed{4} \\ 3 \end{bmatrix}$$

(b) $B = NR$: prob of getting absorbed in one of absorbing states starting from one of transient states

(2) First show that the chain corresponds to regular M.c. as $P^2 > 0$

then find the fixed prob vector using $wP = w$

(3)
$$\left[\begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ 1 & 7 & -6 & 6 \\ 0 & 3 & q & t \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ 0 & 3 & -4 & 5 \\ 0 & 3 & q & t \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ 0 & 3 & -4 & 5 \\ 0 & 0 & q+4 & t-5 \end{array}\right]$$

(a) Singular if $q+4=0$ i.e. there is no third pivot.

(b) $t=5 \Rightarrow$ sol.^{n} exists.

$t \neq 5 \rightarrow$ No sol.^{n} exists.